THERMAL SIMILARITY STUDY
OF A TYPICAL SPACE VEHICLE ELEMENT
IN A CONDUCTING AND RADIATING MODE

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NOTICES

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FOREWORD

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ABSTRACT

Rules of similarity analysis are applied to develop criteria for deducing from model experiments steady-state and transient temperature distributions for a typical space vehicle element exposed to space conditions. The specific purpose is to determine if useful information can be obtained on models in space environment chambers. Thermal similarity criteria are deduced from expressions for transient conduction with radiation boundary conditions. Based on these criteria, a prototype and nominally one-half-scale model were designed and tested in a space chamber. To attain thermal similarity, some geometric distortion was necessary in the model. Both prototype and model were thermally cycled. Temperature data for prototype and model are compared at equivalent locations and times. Results show that temperatures agree within an average of approximately 1 percent with a maximum deviation of approximately 5 percent. The model was subsequently altered, thus destroying its thermal similarity. After alteration, the temperature data differed significantly from that of the thermally similar model and prototype. These experimental results demonstrate the correctness of the derived criteria and show that close adherence to similarity rules is required.

CONTENTS

ILLUSTRATIONS

Figure

NOMENCLATURE

a	Total absorptivity
c	Specific heat
F	Radiation shape factor
k	Thermal conductivity
L	Length of conducting rod
L_{z1}	Thickness of heater plate
L_{z2}	Thickness of cold plate
Q	Total power input to heater plate
Q'	Power input to heater plate per unit volume
R	Radius of heater plate and cold plate
R_e^*	Effective radius of model conducting rod
R_r	Radius of conducting rod
r	Coordinate measured perpendicular to test article axis
r_1^*	Outside radius of model conducting rod based on similarity parameters
r_2^*	Inside radius of model conducting rod based on similarity parameters
T	Absolute temperature
t	Time
t_c	Time length of cycle
V	Volume
z	Coordinate measured along test article axis
α	Thermal diffusivity
β	Inclination angle
ϵ	Total emissivity
ρ	Density
σ	Stefan-Boltzmann constant
ω	Solid angle

SUBSCRIPTS

i	Local value at ith element
j	Local value at jth element
o	Indicates initial condition

SUPERSCRIPTS

$*$	Indicates value for model
$-$	Indicates dimensionless value

SECTION I
INTRODUCTION

As larger booster rockets become available, the size of spacecraft will increase and place severe limitations upon the testing that can be performed in existing space simulation chambers. Although chambers of up to 200 ft in diameter may be feasible (Ref. 1), the growth rate of test chambers has been no match for that of spacecraft. With the development of large spacecraft, the possibility of obtaining valid data by testing small scale models in existing chambers needs to be investigated. In utilizing present chambers thermal similitude appears to be an attractive testing technique. Thermal similitude is the phenomena of two systems obeying the same set of heat-transfer equations and boundary conditions. The values of all dimensionless parameters in these equations and boundary conditions are equal for similar systems.

In addition to being of value for testing, thermal similitude is a useful tool for design. Theoretical design analysis of thermal performance can be experimentally verified by thermally scaling the designed component. Thermal scaling may be to either smaller or larger model sizes as convenience demands. Where analysis is not possible, design parameters may be obtained by thermally testing scale models.

Scaling to a larger size may be desirable when the component to be tested is so small that thermal losses through instrumentation leads and support structure are a significant portion of the total energy transferred to or from the component. To minimize these losses thermal similarity offers a method whereby the component may be increased to a more convenient scale.

In recent years an increasing amount of study has been done on the thermal modeling of spacecraft and spacecraft components (Refs. 2 through 12). However, little experimental data have been published to date. Fowle, Gabron, and Vickers (Ref. 4) report the results of an experimental study in which the temperatures of a prototype were predicted by a scale model* with an accuracy of 3 percent for a one-fifth-scale model and 1 percent for one-half-scale models. Clark, and Clark and Laband performed experiments on space station wall constructions to verify wall temperatures predicted by a theoretical analysis. Comparison between model temperatures and theoretical values was unsatisfactory (Refs. 5 and 6).

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*For convenience "prototype" will hereafter refer to the full-scale test object and "model" will refer to its scale model. Note that the model may be larger or smaller than the prototype.

It is the purpose of this study (1) to review the thermal modeling laws, expressed as a set of similarity ratios, relating the thermal properties of a particular prototype, having mutually reflecting elements, coupled by an irradiated conducting path, to its scaled model, and (2) to verify by an experimental study the validity of these laws as applied to the prototype and model. This study was the subject of a Masters thesis by Shanklin (Ref. 12).

SECTION II
ANALYTICAL PROCEDURES

2.1 FORMULATION OF GOVERNING EQUATIONS

The formulation of equations from which thermal similarity parameters are derived is best implemented by making an energy balance on an incremental volume element of the prototype configuration (e. g. , Refs. 9, 10, 11, and 12). The resulting equation can be used to describe the temperature distribution for the prototype, and it is adequate for the purposes of similarity analysis.

The geometry of the prototype selected for study is shown in Fig. 1. The configuration is constructed in three sections: a heater plate with an internal energy source, a cold plate having no internal energy source, and a solid cylindrical conducting rod connecting both plates. The prototype was fabricated from an isotropic, homogeneous material with gray radiating surfaces. Each section (heater plate, cold plate, and conducting rod) is of uniform thickness. Constant thermal conductivity and heat capacity are also assumed. Because the plate thicknesses are assumed small compared to their diameters, the radiation from the plate edges is considered negligible. Because of symmetry, a one-dimensional, time-dependent temperature field is assumed in each section. The prototype is located in a high vacuum environment, considered to be black and at a radiating temperature so low that the emitted flux is negligible compared to fluxes emitted from the prototype.

2.1.1 Heater-Plate Equation

For application to the heater-plate section of the prototype, the following equation is useful for deriving similarity parameters. A detailed derivation is given in Ref. 12.

$$(1)$$

$$\frac{\partial T_i}{\partial t} = a_i \left(\frac{\partial^2 T_i}{\partial r_i^2} + \frac{1}{r_i} \frac{\partial T_i}{\partial r_i} \right) - \frac{2 a_i \epsilon_i \sigma T_i^4}{k_i L_{zi}} + \frac{a_i}{k_i L_{zi}} \sum_j a_j \epsilon_j F_{i-j} \sigma T_j^4 + \frac{Q_i' a_i}{k_i}$$

Appropriate initial and boundary conditions for the heater plate are

at $t = 0$, $0 \le r_i \le R$, $T_i = T_{oi}$

at $t > 0$, $r_i = R_r$, $k_i (2\pi R_r) L_{z_1} \left(\dfrac{\partial T_i}{\partial r_i}\right)_{r_1 = R_r} = k_i (\pi R_r^2) \left(\dfrac{\partial T_1}{\partial z_i}\right)_{z_1 = 0}$ (2)

at $t > 0$, $r_i = R$, $\left(\dfrac{\partial T_1}{\partial r_i}\right)_{r_i = R} = 0$

2.1.2 Cold-Plate Equation

An equation of the form of Eq. (1) may be used for derivation of similarity parameters for the cold plate. The energy source term must be omitted and the notation changed to reflect the cold-plate configuration. This gives

$$\frac{\partial T_1}{\partial t} = \alpha_i \left(\frac{\partial^2 T_1}{\partial r_i^2} + \frac{1}{r_i} \frac{\partial T_1}{\partial r_i}\right) - \frac{2\alpha_i \epsilon_i \sigma T_1^4}{k_i L_{22}} - \frac{\alpha_1}{k_i L_{22}} \sum_j \alpha_i \epsilon_j F_{i-j} \sigma T_j^4 \quad (3)$$

Initial and boundary conditions for the cold plate are

at $t = 0$, $0 \le r_i \le R$, $T_i = T_{oi}$

at $t > 0$, $r_i = R_r$, $k_i (2\pi R_r) L_{z_1} \left(\dfrac{\partial T_i}{\partial r_1}\right)_{r_1 = R_r} = k_i (\pi R_r^2) \left(\dfrac{\partial T_i}{\partial z_i}\right)_{z_i = L}$

at $t > 0$, $r_i = R$, $\left(\dfrac{\partial T_1}{\partial r_i}\right)_{r_i = R} = 0$ (4)

2.1.3 Cylindrical Conducting Rod Equation

For application to the cylindrical conducting rod section of the prototype, the following equation is used for deriving similarity parameters. A detailed derivation is presented in Ref. 12

$$\frac{\partial T_1}{\partial t} = \alpha_i \left(\frac{\partial^2 T_i}{\partial z_i^2}\right) - \frac{\alpha_i \epsilon_i \sigma T_1^4}{k_i (\frac{1}{2} R_r)} + \frac{\alpha_1}{k_i (\frac{1}{2} R_r)} \sum_j \alpha_i \epsilon_j F_{i-j} \sigma T_j^4 \quad (5)$$

The appropriate initial and boundary conditions for the rod are

at $t = 0$, $0 \le z_i \le L$, $T_i = T_{oi}$

at $t > 0$, $z_i = 0$

$k_1 (\pi R_r^2) \left(\dfrac{\partial T_1}{\partial z_1}\right)_{z_i = 0} = k_i (2\pi R_r) L_{z_1} \left(\dfrac{\partial T_i}{\partial r_1}\right)_{r_i = R_r}$: Heater Plate

at $t > 0$, $z_i = L$

$k_i (\pi R_r^2) \left(\dfrac{\partial T_1}{\partial z_1}\right)_{z_1 = L} = k_i (2\pi R_r) L_{z_2} \left(\dfrac{\partial T_j}{\partial r_1}\right)_{r_1 = R_r}$: Cold Plate (6)

Equations (1), (3), and (5) together with their initial and boundary conditions, Eqs. (2), (4), and (6) may be used to define the similarity between the prototype and model under the listed assumptions. There will be some error in the temperature distribution where the conducting rod is joined to the plates because the conduction is no longer one-dimensional. However, analytical solutions to the above equations are not sought, only the prescription of conditions from which similarity parameters may be obtained. It will be demonstrated that the foregoing equations together with their initial and boundary conditions are suitable for these purposes.

2.2 DERIVATION OF GENERAL SIMILARITY PARAMETERS

In order to design and fabricate a thermally similar model of the prototype configuration, it is necessary to determine the number and form of the parameters governing the similarity between the two configurations. The method to be applied assumes the applicability of a governing equation to a class of problems to which both prototype and model are assumed to belong. The variables are transformed to non-dimensional form, and the governing parameters are obtained by normalizing the equation to nondimensional form. It is a well known method and is discussed in several references (Refs. 13, 14, 15, and 16).

Applying this method to Eq. (1), the following dimensionless variables are defined.

$$\bar{T}_i = \frac{T_i}{T_{oi}}, \quad \bar{r}_i = \frac{r_i}{R}, \quad \bar{z}_i = \frac{z_i}{L}, \quad \bar{t} = \frac{a_i t}{R^2} \tag{7}$$

Substituting these variables into Eq. (1) and normalizing yields the following equation

$$\tag{8}$$

$$\frac{\partial \bar{T}_i}{\partial \bar{t}} = \left(\frac{\partial^2 \bar{T}_i}{\partial \bar{r}_i^2} + \frac{1}{\bar{r}_i} \frac{\partial \bar{T}_i}{\partial \bar{r}_i} \right) - \frac{2 R^2 \epsilon_i \sigma T_{oi}^3}{k_i L_{zi}} \bar{T}_i^4 + \frac{R^2}{k_i L_{zi} T_{oi}} \sum_j a_i \epsilon_j F_{i-j} \sigma T_j^4 + \frac{Q_i' R^2}{k_i T_{oi}}$$

Substituting the dimensionless variables, Eq. (7), into the initial and boundary conditions for the heater plate, Eq. (2), and normalizing yields the following dimensionless initial and boundary conditions:

at $\bar{t} = 0$, $0 \le \bar{r}_i \le 1$, $\bar{T}_i = 1$

at $\bar{t} > 0$, $\bar{r}_i = \frac{R_r}{R}$, $\left(\frac{\partial \bar{T}_i}{\partial \bar{r}_i} \right)_{\bar{r}_i = \frac{R_r}{R}} = \frac{\frac{R_r}{2} R}{L_{zi} L} \left(\frac{\partial \bar{T}_i}{\partial \bar{z}_i} \right)_{\bar{z}_i = 0}$ $\tag{9}$

at $\bar{t} > 0$, $\bar{r}_i = 1$, $\left(\frac{\partial \bar{T}_i}{\partial \bar{r}_i} \right)_{\bar{r}_i = 1} = 0$

A similar group of dimensionless equations with their associated initial and boundary conditions can be written for a nominally geometrically similar heater plate, similarly located with respect to its environment. If thermal similarity is to exist between the prototype and model, the dimensionless equations must be identical. If the prototype and model are to behave in a similar manner, their dimensionless initial and boundary conditions must also be identical. For the identity of the equations and for solutions to the equations to be the same at equivalent points, the following similarity criteria must be maintained.

$$\frac{R^{*2} \epsilon_i^* \sigma T_{oi}^{*3}}{k_i^* L_z^*} = \frac{R^2 \epsilon_i \sigma T_{oi}^3}{k_i L_z} \tag{10a}$$

$$\frac{R^{*2} a_i^* \epsilon_j^* F_{i-j}^* \sigma T_j^{*4}}{k_i^* L_z^* T_{oi}^*} = \frac{R^2 a_i \epsilon_j F_{i-j} \sigma T_j^4}{k_i L_z T_{oi}} \tag{10b}$$

$$\frac{Q_i'^* R^{*2}}{k_i^* T_{oi}^*} = \frac{Q_i' R^2}{k_i T_{oi}} \tag{10c}$$

$$\frac{R_r^*}{R^*} = \frac{R_r}{R} \tag{10d}$$

$$\frac{R_r^* R^*}{L_z^* L^*} = \frac{R_r R}{L_z L} \tag{10e}$$

The initial condition and the second boundary condition in Eq. (9) give no useful information that can be used in the similarity analysis.

A similar analysis applied to Eqs. (3) and (4) yields identical results for the cold plate with the following exceptions: (1) the internal energy term, Eq. (10c), is omitted, and (2) the notation for plate thickness is changed from L_{z1} to L_{z2}. Because of this identity of results, the subscripts, 1 and 2, have been omitted from the plate thickness terms of Eq. (10) for the sake of generality. The term L_z will hereinafter be used in the similarity ratios to denote both the heater-plate and cold-plate thicknesses.

An analysis of Eqs. (5) and (6) yields the following scaling criteria for the conducting rod.

$$\frac{L^{*2} \epsilon_i^* \sigma T_{oi}^{*3}}{k_i^* R_r^*} = \frac{L^2 \epsilon_i \sigma T_{oi}^3}{k_i R_r} \tag{11a}$$

$$\frac{L^{*2} a_i^* \epsilon_j^* F_{i-j}^* \sigma T_j^{*4}}{k_i^* R_r^* T_{oi}^*} = \frac{L^2 a_i \epsilon_j F_{i-j} \sigma T_j^2}{k_i R_r T_{oi}} \tag{11b}$$

$$\frac{R_r{}^* R^*}{L_z{}^* L^*} = \frac{R_r R}{L_z L} \tag{11c}$$

The ratios listed in Eqs. (10) and (11) are the general similarity parameters used to describe the similarity between prototype and model. Since the dimensionless equations are assumed to be applicable to both prototype and model, it follows that equivalent points in both configurations will exhibit similar behavior under the conditions assumed.

2.3 DESIGN OF MODEL

The similarity parameters having been obtained, it remains to apply them to the design of a model. Material, surface properties, and temperature are to be preserved from prototype to model. The surfaces are assumed to be blackbody radiators. The model is to be designed to a nominal one-half scale, although geometric distortion will be permitted.

Applying the equality of material, surface properties, and temperature to the similarity ratios obtained for the heater plate reduces them to the following criteria:

$$\frac{R^{*2}}{L_z{}^*} = \frac{R^2}{L_z} \qquad \text{(From Eq. 10a)} \tag{12a}$$

$$\frac{R^{*2} F_{i-j}{}^*}{L_z{}^*} = \frac{R^2 F_{i-j}}{L_z} \qquad \text{(From Eq. 10b)} \tag{12b}$$

$$Q'^* R^{*2} = Q' R^2 \qquad \text{(From Eq. 10c)} \tag{12c}$$

$$\frac{R_r{}^*}{R^*} = \frac{R_r}{R} \qquad \text{(From Eq. 10d)} \tag{12d}$$

$$\frac{R_r{}^* R^*}{L_z{}^* L^*} = \frac{R_r R}{L_z L} \qquad \text{(From Eq. 10e)} \tag{12e}$$

Similarly for the conducting rod, the similarity ratios become

$$\frac{L^{*2}}{R_r{}^*} = \frac{L^2}{R_r} \qquad \text{(From Eq. 11a)} \tag{13a}$$

$$\frac{L^{*2} F_{i-j}{}^*}{R_f{}^*} = \frac{L^2 F_{i-j}}{R_f} \qquad \text{(From Eq. 11b)} \tag{13b}$$

$$\frac{R_r{}^* R^*}{L_z{}^* L^*} = \frac{R_r R}{L_z L} \qquad \text{(From Eq. 11c)} \tag{13c}$$

It is immediately seen that the first and second ratios in each group require the identity of the shape factors, F_{i-j}, for prototype and model. The expression for the shape factor of a surface element, dA_i, with respect to a finite surface A_j is (Ref. 16)

$$F_{i-j} = \frac{1}{\pi} \int \cos \beta_i \, d\omega_i \qquad (14)$$

where ω_i is the solid angle subtended by A_j as viewed from dA_i, and β_i is the angle between the normal to dA_i and the line of sight to a differential area of A_j. It is seen that if the surfaces that can see each other and the distance between these surfaces are scaled geometrically, the shape factor remains the same from model to prototype. Because one-half was selected as the nominal modeling scale, Eqs. (12a) and (13a) require that

$$R^* = \tfrac{1}{2}R \qquad \text{(From Eq. 12a)} \qquad (15a)$$

$$L^* = \tfrac{1}{2}L \qquad \text{(From Eq. 13a)} \qquad (15b)$$

and thus

$$L_z^* = \tfrac{1}{4}L_z \qquad \text{(From Eq. 12a)} \qquad (15c)$$

$$R_r^* = \tfrac{1}{4}R_r \qquad \text{(From Eq. 13a)} \qquad (15d)$$

The requirements of Eqs. (12e) and (13c) are fulfilled by the model dimensions defined in Eq. (15).

Equation (15c) requires that the thickness of both model plates be scaled one-fourth rather than in the geometric scale of one-half. This distortion causes no real difficulty since the radiation from the edge of the plates has already been assumed to be small compared to the total plate radiation. Because the edges of the plates do not see any other part of the configuration, they do not influence the shape factor of any other section of the model.

Let the total energy input to the heater plate be defined by

$$Q = Q'V \qquad (16)$$

where the heater-plate volume is equal to $\pi R^2 L_{z1}$. Equation (12c) requires that Q'^* be four times Q', but the geometry of the scaled heater plate requires the model volume, V^*, to be one-sixteenth the volume of the prototype heater plate. Thus the total power delivered to the model heater plate is prescribed by

$$Q^* = \tfrac{1}{4}Q \qquad (17)$$

Equation (12d) requires the model conduction rod radius to be reduced by the nominal scale factor of one-half. This radius has previously been determined to be one-fourth the prototype rod radius by Eq. (15d). If, however, one-fourth scale is used, then not only the requirements of Eq. (12d) are not fulfilled, but the model shape factors are not equal to the prototype shape factors. If one-half scale is used, then not only the requirement of Eq. (15d) is not fulfilled, but also those of Eqs. (12e) and (13c). These difficulties are resolved by the following device. Define an "effective" radius, R_e*, by

$$R_e{}^* = \frac{r_1{}^{*2} - r_2{}^{*2}}{r_1{}^*} \tag{18}$$

where r_1* is the external radius of the model rod defined by

$$r_1{}^* = \tfrac{1}{2} R_r \tag{19}$$

and r_2* is the inside radius of the rod, bored to give a value of R_e* defined by

$$R_e{}^* = R_r{}^* = \tfrac{1}{4} R_r \tag{20}$$

If R_e* is substituted for R_r* in the similarity ratios requiring the rod radius to be reduced by one-fourth, and if r_1* is substituted for R_r* in the similarity ratios requiring the rod radius to be reduced by one-half, the geometric requirements of the similarity ratios will be satisfied. In addition, the shape factor equality is maintained because the outside radius is geometrically scaled.

The following geometric requirements have been established for the fabrication of a thermally similar model of the prototype shown in Fig. 1:

$$R^* = \tfrac{1}{2} R \tag{15a}$$

$$L^* = \tfrac{1}{2} L \tag{15b}$$

$$L_{z_1}{}^* = \tfrac{1}{4} L_{z_1} \tag{21}$$

$$L_{z_2}{}^* = \tfrac{1}{4} L_{z_2} \tag{22}$$

$$r_1{}^* = \tfrac{1}{2} R_r \tag{19}$$

$$r_2{}^* = \frac{1}{2} \frac{R_r}{2} \tag{23}$$

The geometric configuration of the model, designed to this criteria, is shown in Fig. 2.

The total power supplied to the heater plate of the model was found to be one-fourth that supplied to the prototype heater plate. At this

power for equivalent points on the model and prototype and at equivalent times, the model will behave in a similar manner as the prototype. Equivalent times are defined by Eq. (7), and it is seen that the time required for a thermal change to occur in the model is one-fourth that for the same change to occur in the prototype. The environments in which the two configurations are placed are assumed to be identical.

SECTION III
EXPERIMENTAL EQUIPMENT

3.1 TEST FACILITY

The experimental program was conducted in the Aerospace Research Chamber (5V) of the Aerospace Environmental Facility (AEF) located at the Arnold Engineering Development Center (AEDC). The facility is a 5- by 5- by 13-ft mild carbon steel chamber equipped with a -320°F liner having an area of approximately 250 sq ft of optically black surface. The chamber is equipped with two 20-in. oil diffusion pumps. Base pressures of 10^{-6} torr* are typical.

Both the prototype and model were mounted from cables in the test section. The installed prototype is shown in Fig. 3, and the installed model is shown in Fig. 4.

3.2 PROTOTYPE AND MODEL

3.2.1 Prototype

The prototype tested consisted of two parallel flat plates connected by a cylindrical conducting path. The prototype was constructed in two sections, a "spool" piece and a separate heater. The heater plate was bolted to the spool, and the joint between them filled with a high thermal conductivity cement to eliminate the contact resistance between heater and spool. A sketch of the prototype geometry showing its physical dimensions is shown in Fig. 1.

The heater was constructed by machining a uniform spiral groove in the surface of a plate. The groove was large enough to accommodate a 14-gage, high resistance, electrical heater wire. The heater wire was held in place by an electrical insulating cement. The heater plate

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*One torr is equal to one millimeter of mercury.

was attached to the spool with the grooves facing toward the cell walls. A photograph of the heater is shown in Fig. 5.

The prototype heater plate and spool were fabricated entirely from mild carbon steel plate (ASTM A-7) and round bar stock (AISI 1018 HR). With the exception of the joint between heater plate and spool, all joints were welded.

After installation into the test chamber, all surfaces of the prototype were blackened with a blow torch. The blackening was sufficiently thick to cover all bare metal.

3.2.2 Model

The model tested was a thermally similar, nominally one-half-scale model of the prototype configuration. The model was fabricated in an identical manner as the prototype with the exception of the cylindrical conducting rod. The model conducting path was bored to achieve a cross-sectional area required by the similarity ratios used, in Section II, to design the model. The space within the cylinder was filled with lightly crumpled aluminum foil, providing a shield for radiation transfer within the cylinder while at the same time offering a high resistance, because of the foil's small cross-sectional area, to conduction heat transfer. A sketch of the model configuration showing its physical dimensions is shown in Fig. 2.

The material used for fabrication was identical to that used for the prototype, and welded construction was again employed where metal-to-metal joints were required, with the exception of the joint between the heater plate and spool. The model heater plate was attached to the spool in an identical manner as the prototype, and the same type of high conductivity cement was used to reduce the joint resistance between the heater plate and spool piece.

The heater plate was constructed in the same manner as the prototype. The same type of electrical insulating cement was used to hold a 20-gage electrical resistance heater wire in place. The heater was again attached to the spool with the grooves facing outward. A photograph of the heater plate is shown in Fig. 6.

After installation in the test chamber, all surfaces of the model were blackened in a manner identical to that of the prototype.

3.2.3 Modified Model

Subsequent to tests using the thermally similar model, described above, the model was altered by two different means. In the first alteration the bore of the model conducting rod was increased to 2.25 in., thus increasing the thermal conduction resistance of the rod by approximately 159 percent. In a second and subsequent modification the bore of the model was completely plugged using a mild steel cylinder machined to fit within the 2.25-in.-diam bore of the high resistance rod. A high thermal conductivity cement was used at both ends of the rod to reduce joint resistance between the plug and the heater plate and the plug and the cold plate. The thermal resistance of the rod was thus reduced approximately 50 percent from the original similar model rod.

3.3 PROTOTYPE AND MODEL INSTRUMENTATION

Temperatures at 36 locations on the prototype and model were measured using copper-constantan thermocouples. The thermocouples were attached to the prototype and model surface by peening the junction into the surface. Heat losses were minimized through junctions by attaching the thermocouple leads to the test article surface for several inches from the tips. Losses through the leads were minimized by using small (28-gage) wire and by wrapping wire bundles in aluminum foil to reduce radiation loss.

Although the temperatures at 36 locations were measured on both prototype and model, it is appropriate to use the temperatures at three locations since these temperatures are representative of the temperature levels in the test articles.

The locations of these three thermocouples on the prototype are shown in Fig. 1. The model thermocouple locations correspond to those on the prototype and are located on the model as shown in Fig. 2.

Temperatures at selected locations on the walls of the test chamber were also measured. Copper-constantan thermocouples were used for these measurements. These thermocouples were located to obtain representative chamber wall temperatures and are a permanent part of the test chamber.

3.4 INSTRUMENTATION FOR DATA RECORDING AND MONITORING

3.4.1 Temperature Recording

Using the automatic sampling, scanning, and recording instrument, outputs from the thermocouples located on the prototype, model, and in the chamber walls were printed on a continuous strip chart. A digital clock for monitoring the time and also a timer which allowed data to be taken automatically at 5-min intervals was included. Data could also be taken continuously. A digital scanner which allowed visual monitoring of the thermocouple outputs was also provided. The temperature sampling time for each thermocouple was 1 sec.

3.4.2 Heater Power Measurement and Control

The power input to the heater plates of the prototype and model was visually monitored using a calibrated wattmeter. Alternating current was supplied to the prototype and model heater plates. Power was manually controlled with a variable transformer.

3.4.3 Chamber Pressure

Chamber pressure was visually monitored. An ion gage was placed at each end of the test chamber.

SECTION IV
TEST PROCEDURES, RESULTS, AND ANALYSIS OF DATA

4.1 PROCEDURE

4.1.1 Equilibrium Conditions

The procedure followed in all tests of prototype, model, and modified models was to pump down the test chamber, using the 20-in. diffusion pumps, for approximately 5 hr. At this time the heater plate and cryoliner were energized. The power delivered to the prototype heater plate was 2000 w for each of two tests and 1500 w for each of two additional tests. Corresponding powers of 500 and 375 w were supplied to the model heater plate. The test article was allowed to approach equilibrium, the test chamber walls being maintained at approximately -300°F. Constant power was delivered to the heater plate. The pressure was maintained in the 10^{-5} torr range at all

times. Equilibrium was assumed to have been reached when all proto-
type temperatures rose no more than five degrees per hour and all
model temperatures no more than five degrees in the equivalent time
of 15 min. Temperature data were recorded every 5 min. Heater-
plate power input, test cell pressures, and cell liner temperatures
were continuously monitored.

4.1.2 Thermal Cycling

Steady-state conditions having been approached, the following pro-
cedure was used for all tests of prototype, model, and altered models.
The power to the heater plate was turned off and the test article allowed
to cool: 100 min for the prototype and an equivalent time of 25 min for
the model.

At the end of the cooling period the power to the heater plate was
turned on with the same value as during the initial heat-up to equilibrium.
Power was maintained for 100 min to the prototype heater plate and
25 min to that of the model.

Prototype and model were cycled in this manner through two com-
plete cycles. As herein used, a cycle is defined as one cooling period
plus an equal period of heating. Thus for the prototype a cycle is
200 min in length and for the model, 50 min.

Temperature data were recorded every 5 min for all tests. Heater
plate power input, test cell pressure, and cell liner temperatures were
monitored continuously to ensure uniform cell conditions. Adjustments
in the rate of liquid-nitrogen supply to the cell liner were made as
required. The power input to the heater plate was maintained constant
within ±10 w for the prototype and ±5 w for the model.

4.2 TEST RESULTS AND ANALYSIS OF DATA

The results of this investigation are presented in this section in
graphical form. Prototype and model temperatures at three thermo-
couple locations are compared at reduced times for both high and low
power test runs. Prototype, model, and modified model cold plate
temperatures are compared graphically for high and low power levels.

Figure 7 shows the heater-plate temperatures at thermocouple
location No. 1 for both prototype and model. The temperatures are
plotted at both high and low power input values. The plotted lines
represent prototype data, and the data points represent the model

temperatures and indicate the similarity of thermal behavior between model and prototype.

The heater plate exhibits the least correlation between prototype and model temperatures. Model temperatures deviate from those of the prototype by an average of approximately 1.5 percent. The maximum deviation between the two test article temperatures is of the order of 5 percent of the prototype temperature.

The heater plate was modeled only in regard to the external geometric requirements. No attempt was made to model the heater wire imbedded in the plate, the insulating cement used to hold the heater wire in place, or the joint resistance between the heater plate and spool. The heater plate was modeled as if it were composed on one material entirely. Even though the modeling was inexact, the overall agreement between model and prototype temperatures was excellent.

The conducting rod was modeled using the equivalent radius discussed in Section II. The external geometry was scaled exactly; however, the internal configuration was altered significantly, a solid section being modeled by a tubular section. The correlation of rod temperatures was quite good. The model temperatures duplicated those of the prototype within an average of approximately 1 percent.

Figure 8 presents the rod temperatures at thermocouple location No. 2 for both model and prototype. The temperatures are plotted at high and low power test runs. The lines represent prototype temperatures, and the data points, indicating similarity of temperature behavior between model and prototype, represent the model temperatures.

Figure 9 presents the cold-plate temperatures at thermocouple location No. 3 for both model and prototype. As can be seen, the modeling of the prototype was excellent. Cold-plate temperatures for the prototype were duplcated by the model within an average of 0.7 percent. The cold-plate thermal modeling was exact except at the junction of the rod and the plate, and the cold plate yields a closer agreement between temperatures than any other section of the configuration.

Figures 10 and 11 are scatter diagrams of model cold-plate temperatures for the similar, high resistance, and low resistance rod models. As can be seen, there was a significant difference between the three groups of data for both high and low power test runs. The low resistance rod resulted in higher cold-plate temperatures because conduction is increased through the rod. These deviations from the

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prototype temperatures are less pronounced at higher temperatures and indicate that at these higher temperatures more energy is being transferred to the cold plate by radiation and relatively less conducted through the rod to the cold plate. The high resistance rod resulted in lower cold-plate temperatures because conduction through the rod is decreased. Because of more radiation transfer to the cold plate at higher temperatures and thus relatively less conduction through the rod, the deviations from the prototype are less than at the lower test temperatures.

Figures 10 and 11 also indicate that the modeling of the conducting rod in the similar model also was not exact. The resistance of the rod is slightly less than that required by the similarity parameters. Because of manufacturing tolerances the rod resistance could be less than the exactly scaled rod by approximately 2 percent. In addition, the use of aluminum foil as a radiation shield in the center of the rod could have permitted more conduction than was anticipated in the model design.

In summary, the temperature data of the similar model, designed to conform to the derived thermal similarity criteria, illustrated excellent agreement with the temperature data of the prototype at equivalent locations and times. In further substantiation of the correctness of the similarity criteria, the temperature data of the altered models deviated markedly from that of the prototype. A close examination of Figs. 10 and 11 will show that the qualitative behavior of the altered models also was, in substance, different from that of the similar model.

SECTION V
CONCLUSIONS

The following conclusions are made as a result of the experimental investigation of thermal similarity phenomena:

1. Test data of the similar model, designed to conform to the derived similarity criteria, were in excellent agreement with the prototype data, showing that thermal scaling is possible and establishing the validity of the derived criteria.

2. Geometric distortion may be successfully used as a modeling technique, care being taken not to alter the radiation shape factors.

3. Test data of the altered models differed significantly from those of the prototype and further substantiated the correctness of the scaling criteria and the method used in their application.

4. Useful information can be obtained from models in small space environment chambers if the criteria of thermal similarity are approximated.

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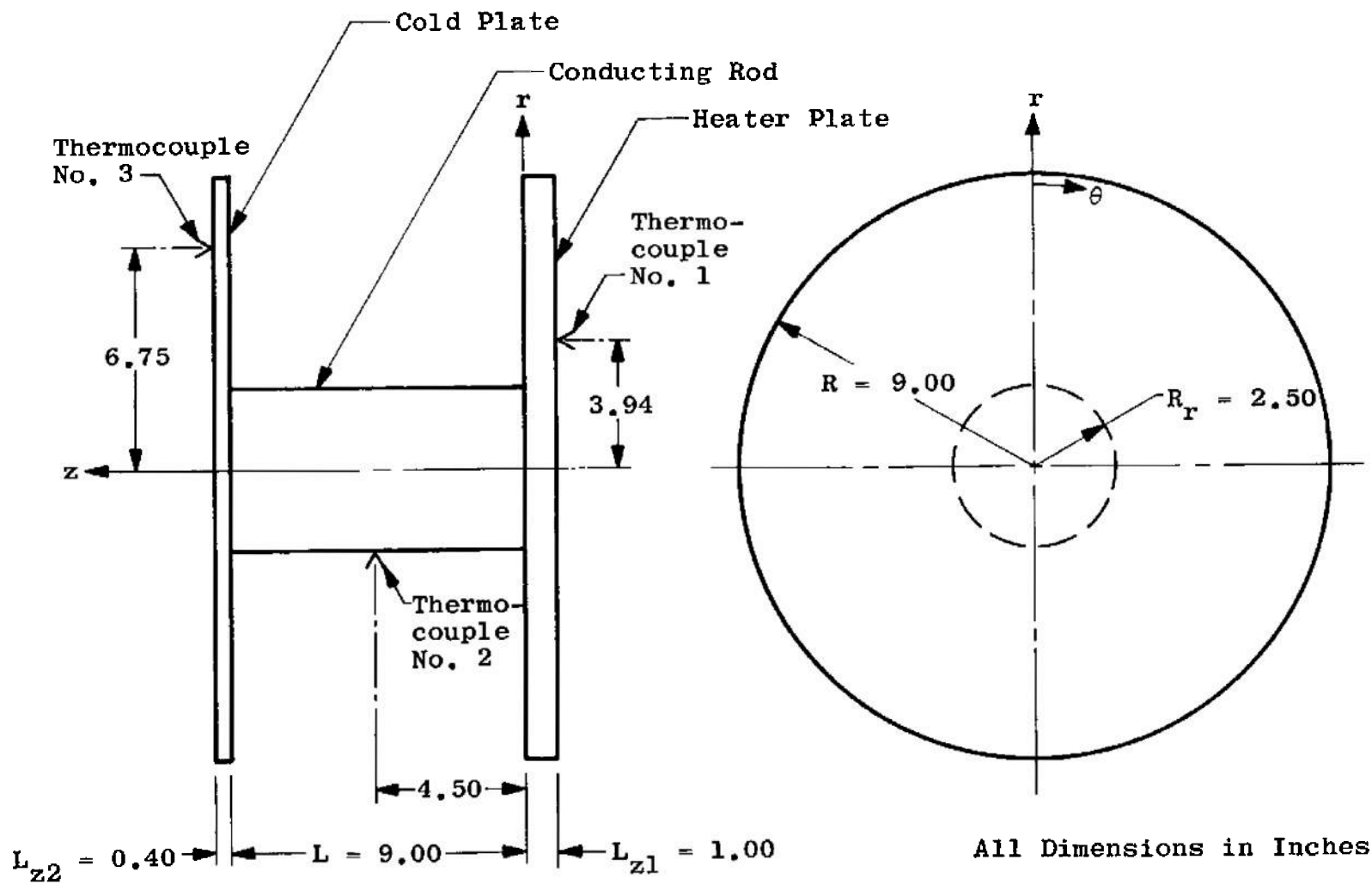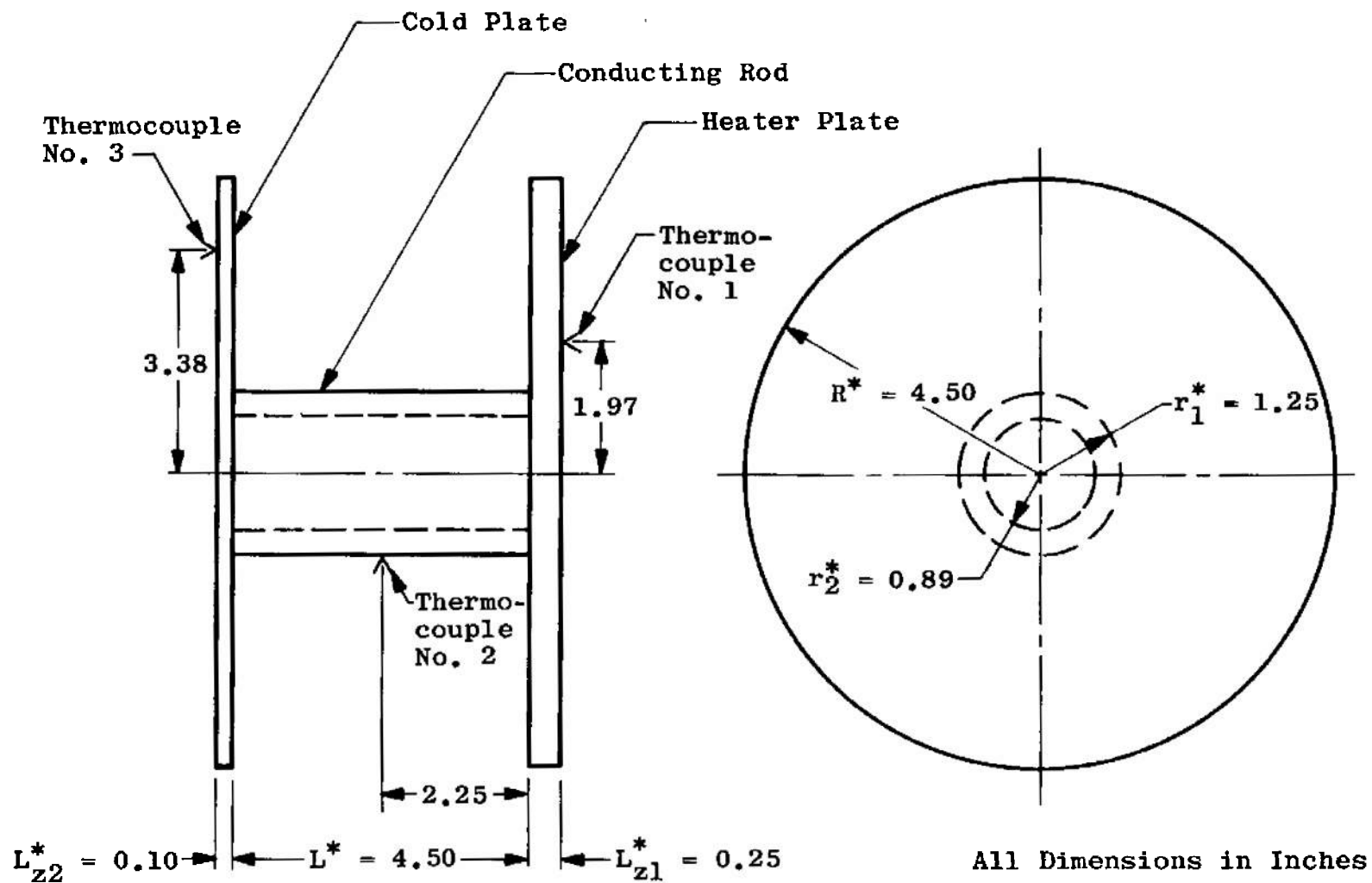
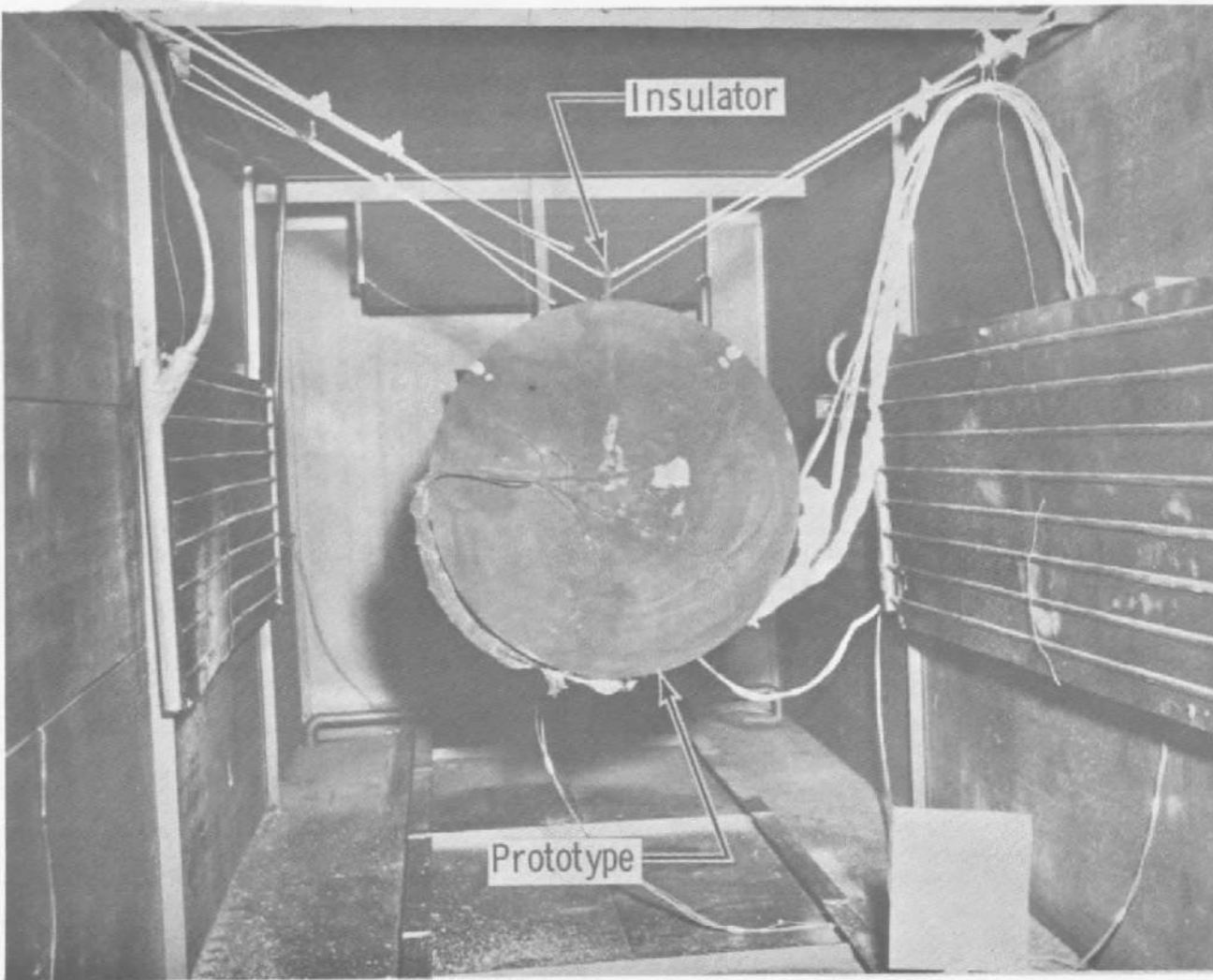Fig. 1 Prototype Geometry

Fig. 2 Model Geometry
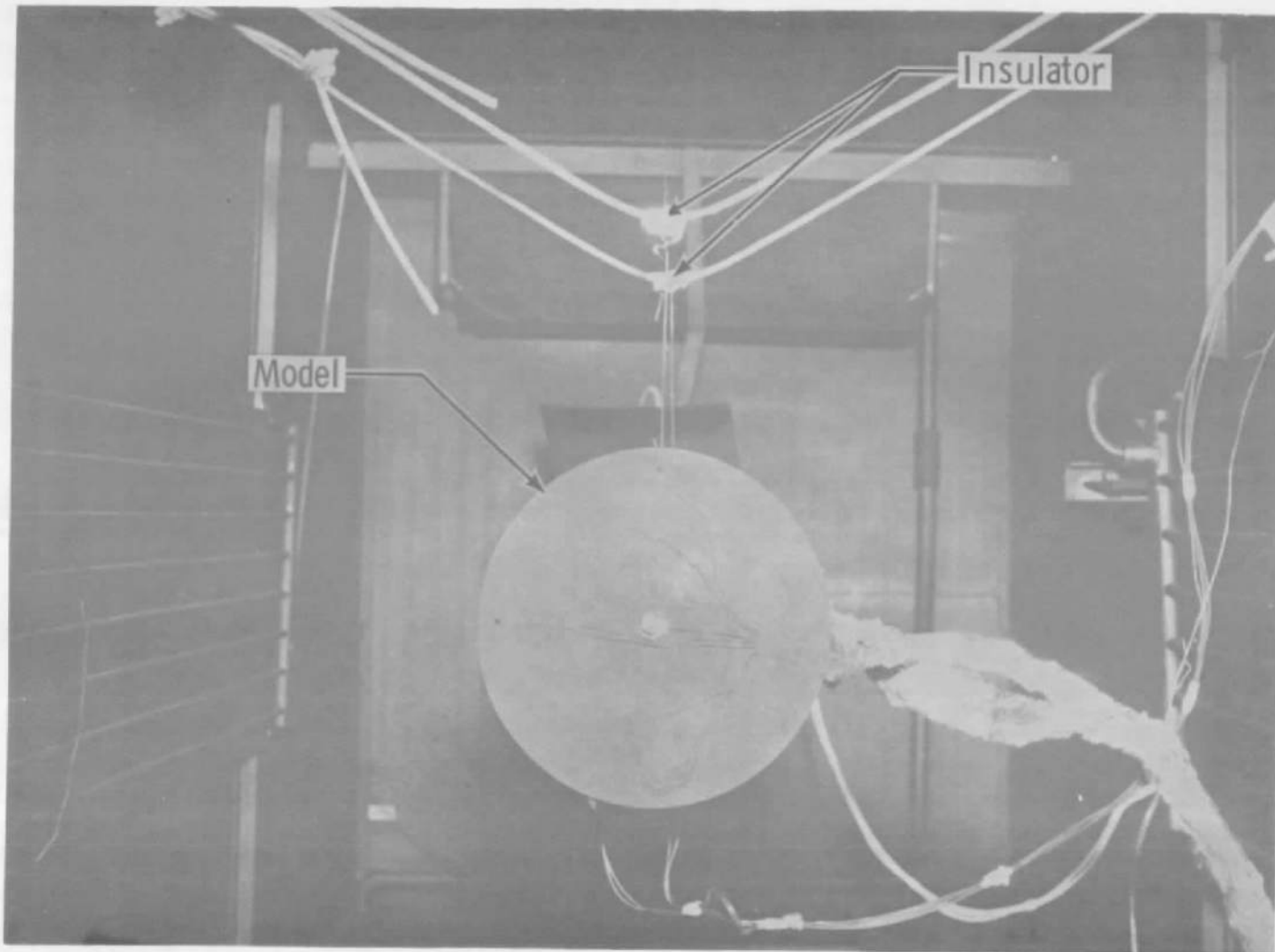
Fig. 3 Prototype Installation
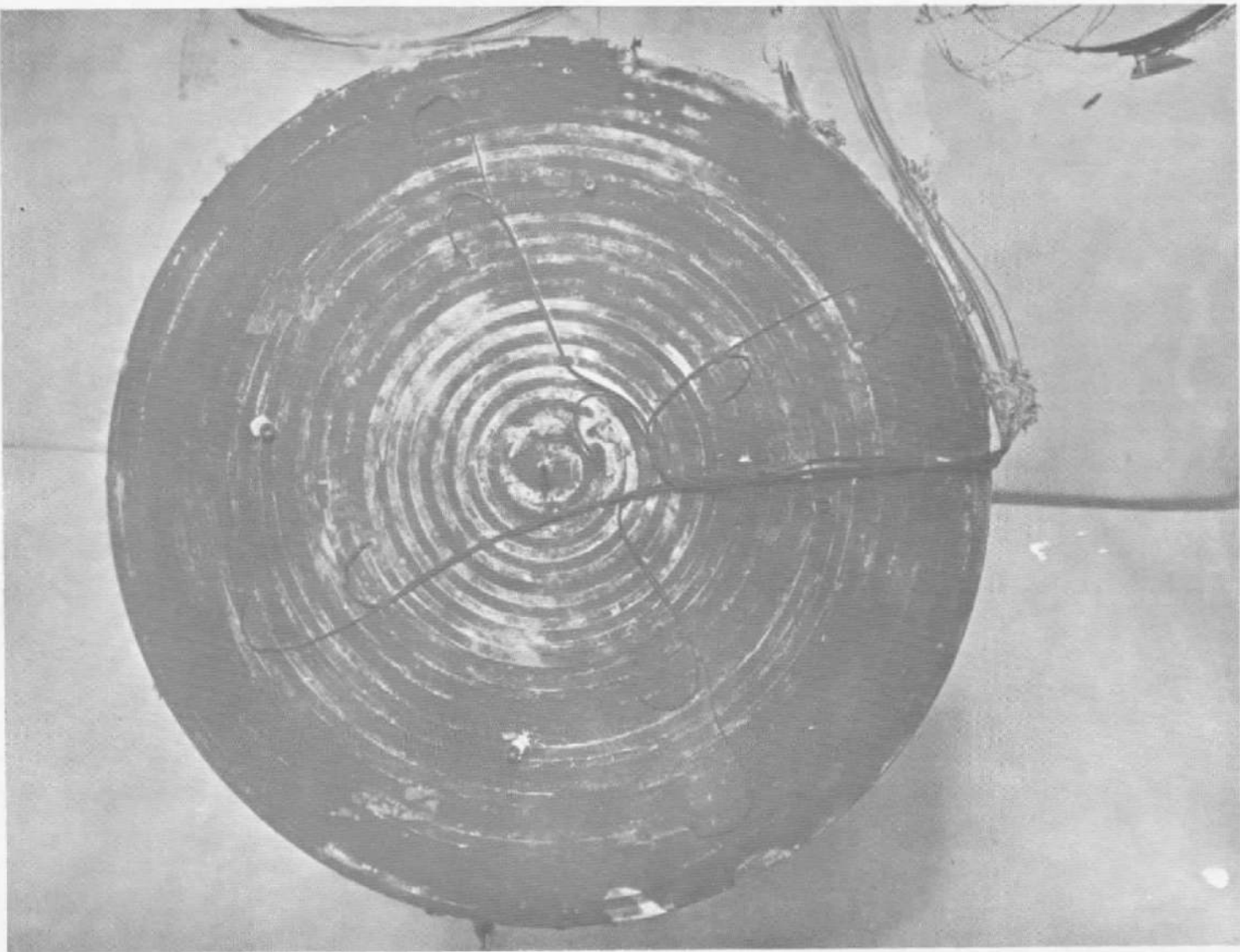
Fig. 4 Model Installation

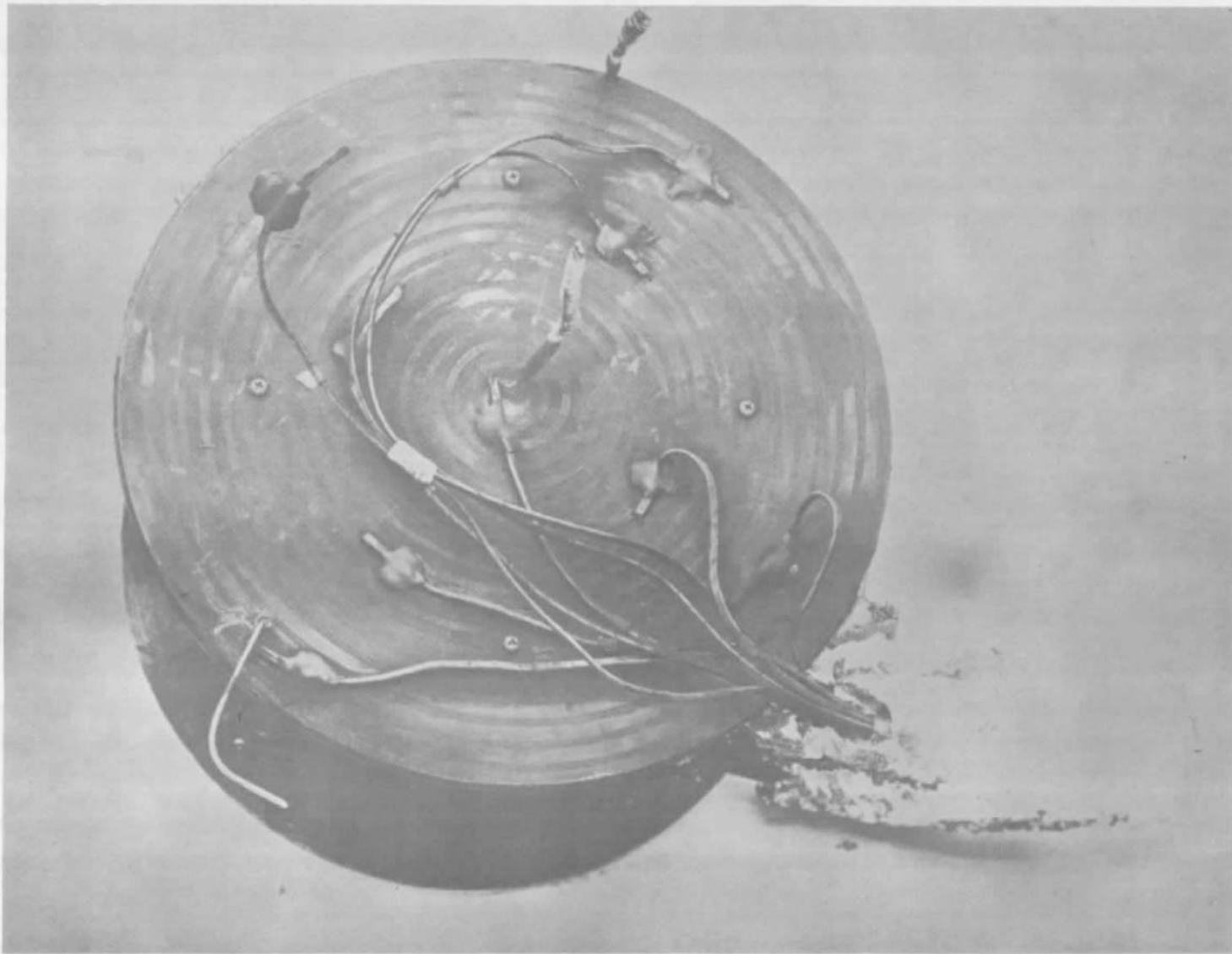Fig. 5 Prototype Heater Showing Grooved Construction
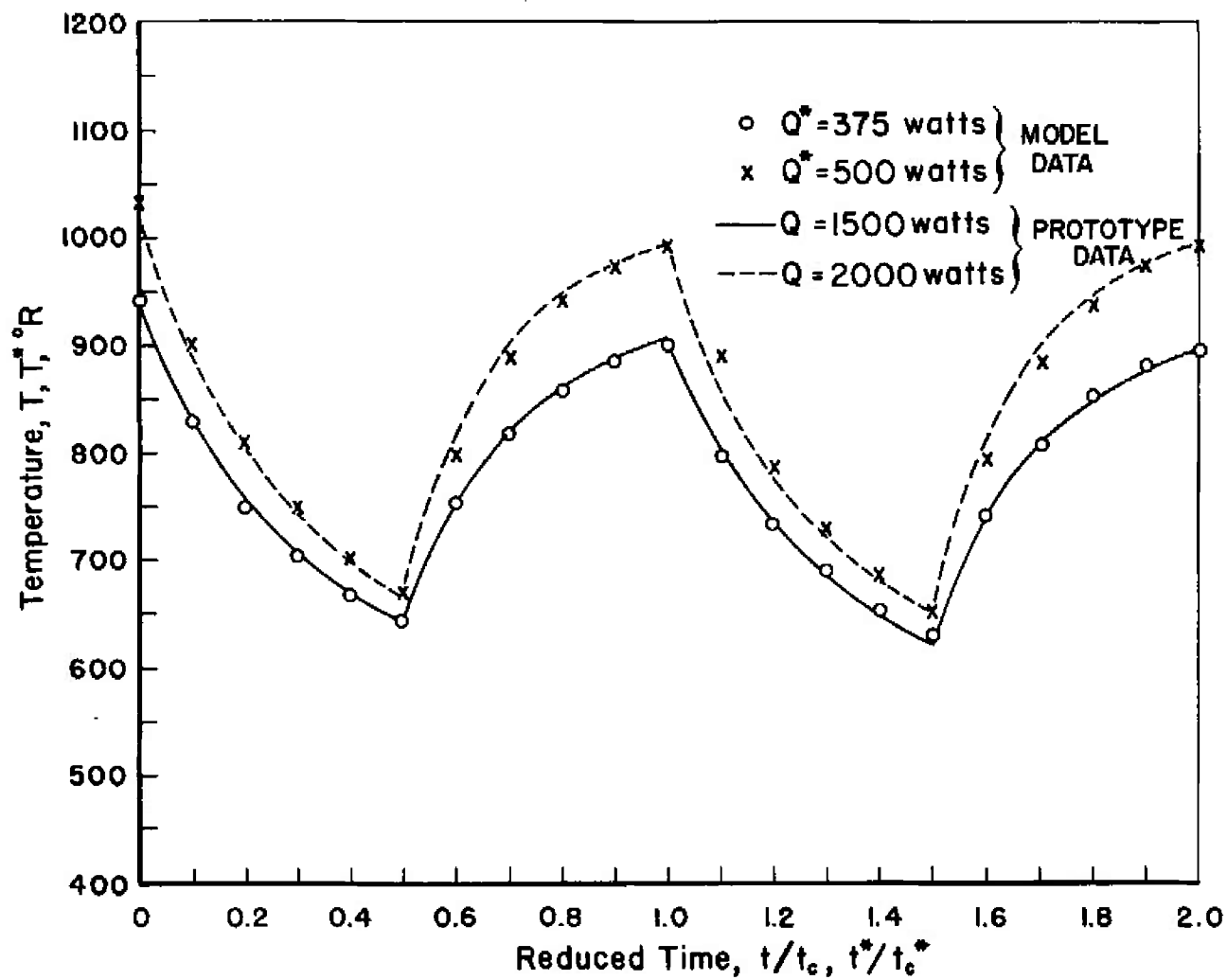
Fig. 6 Model Heater Showing Grooved Construction

Fig. 7 Time Variation of Prototype and Model Temperatures at Thermocouple Location No. 1
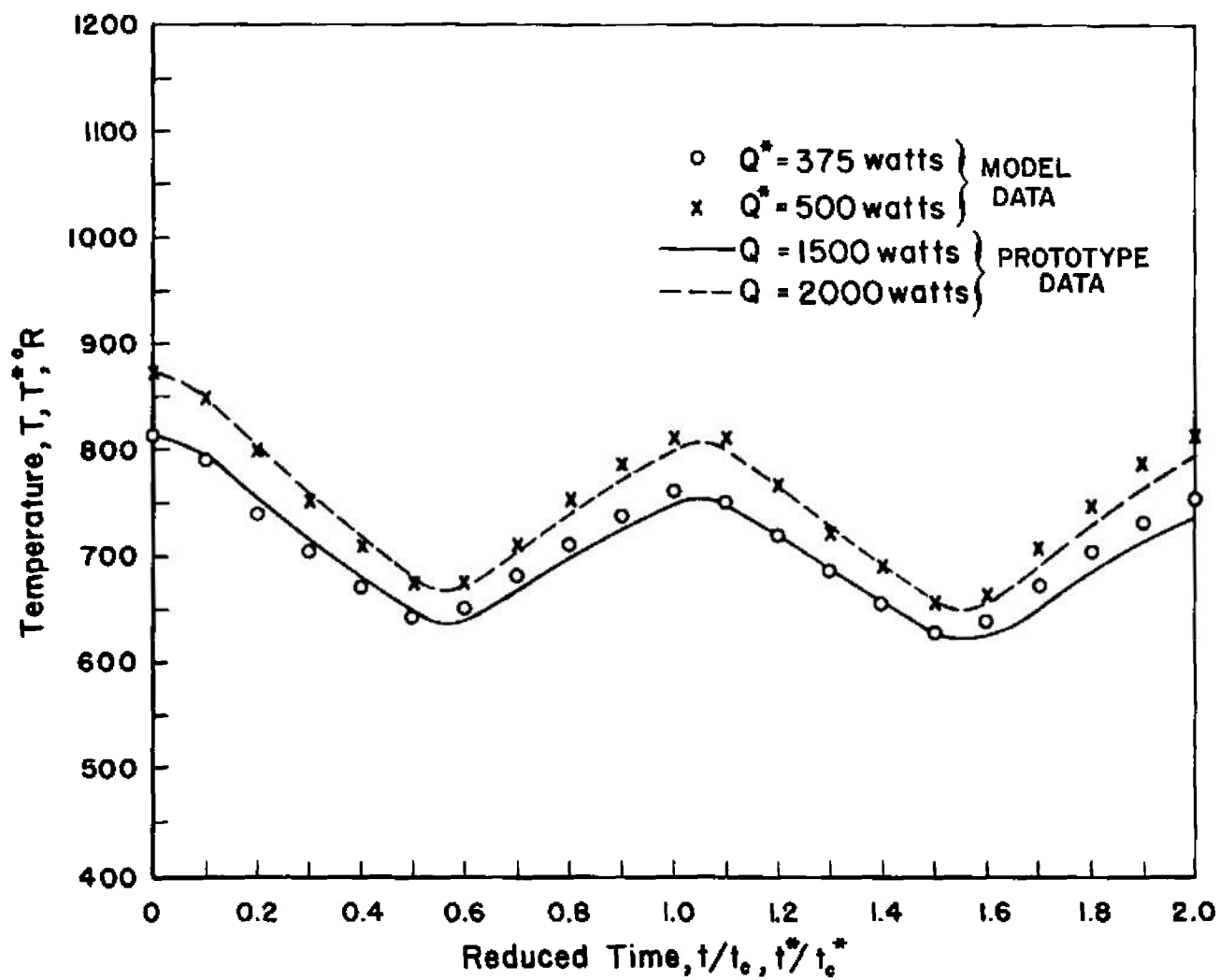
Fig. 8 Time Variation of Prototype and Model Temperatures at Thermocouple Location No. 2
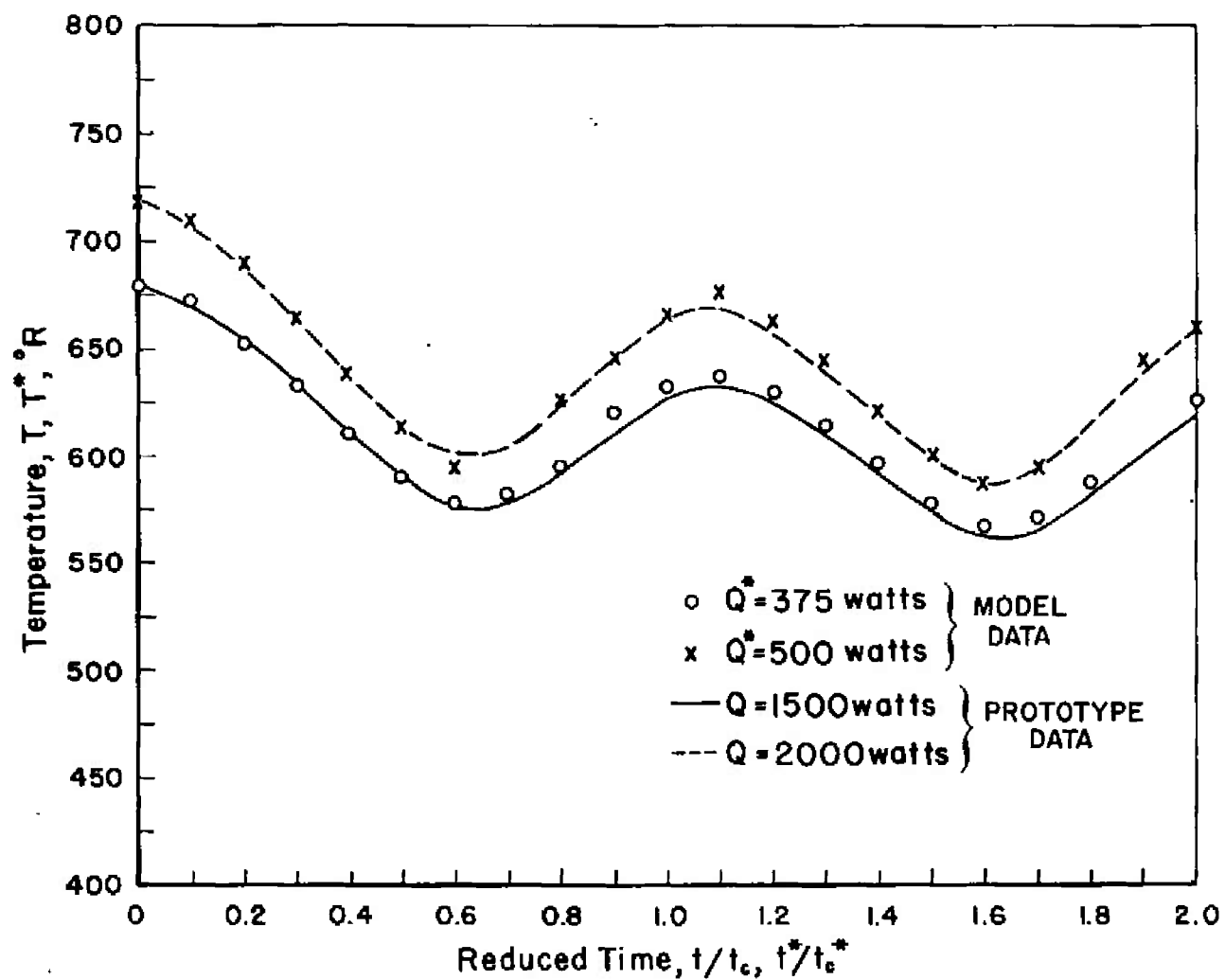
Fig. 9 Time Variation of Prototype and Model Temperatures at Thermocouple Location No. 3

Fig. 10 Comparison of Model Temperatures with Corresponding Prototype Temperatures at Thermocouple Location No. 3 and with Heater Power Input of 375 watts
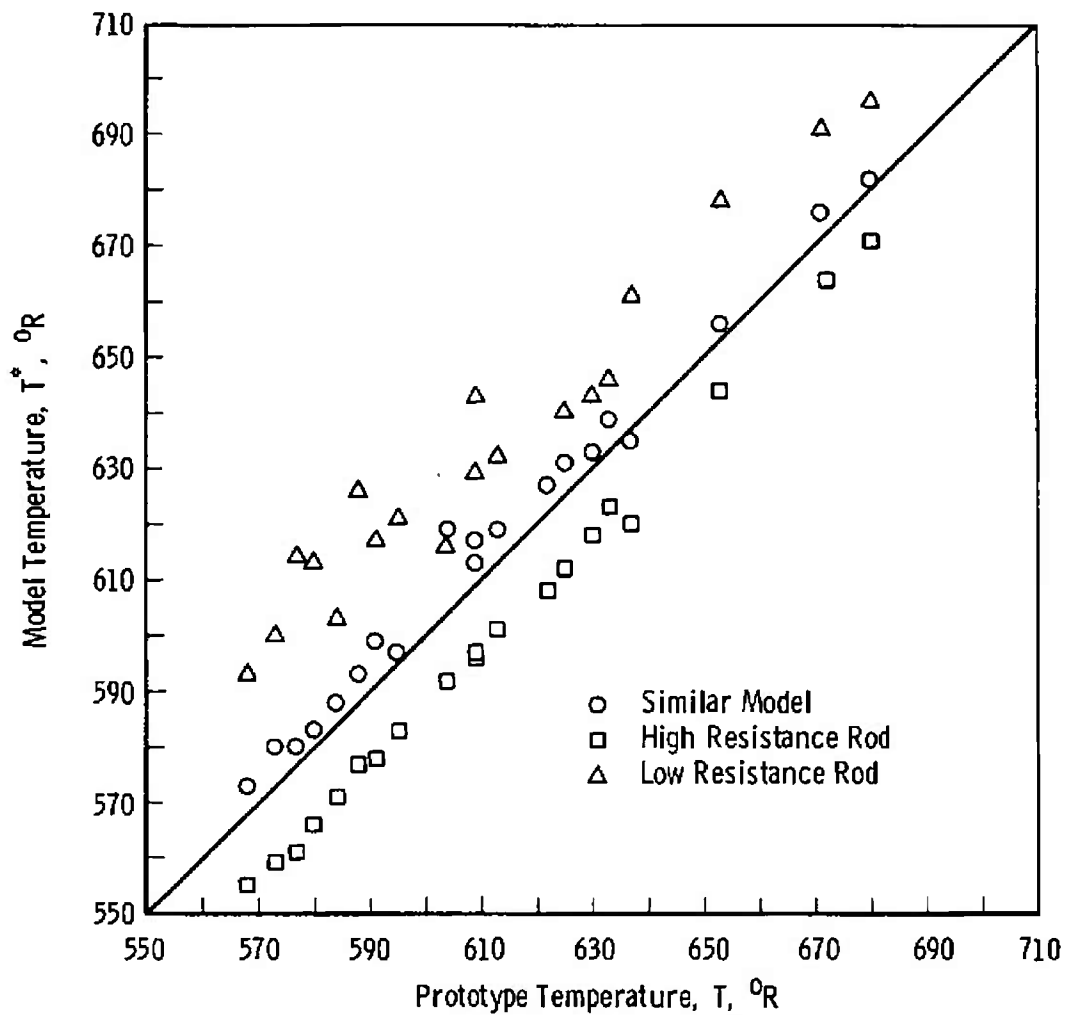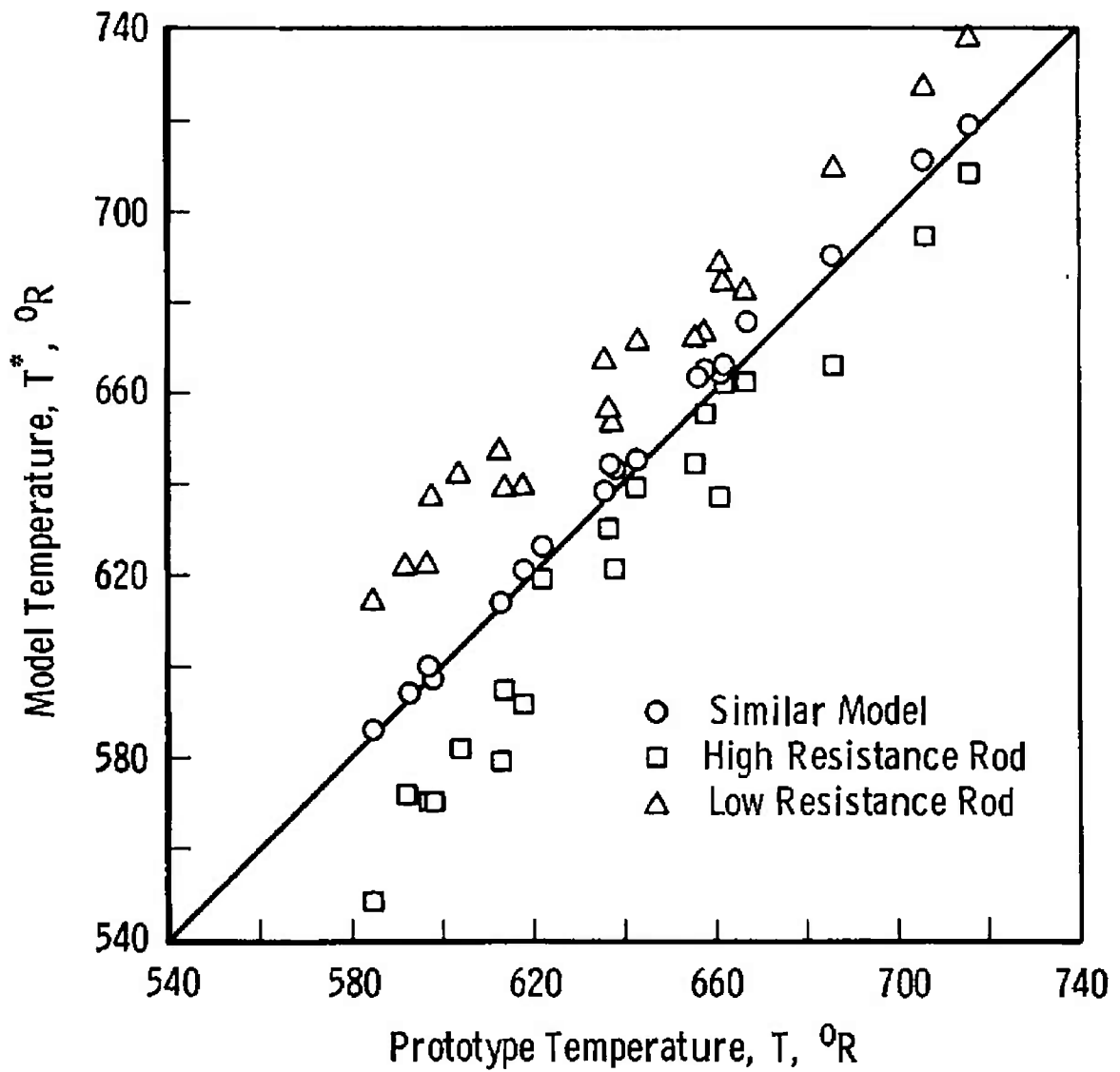
Fig. 11 Comparison of Model Temperatures with Corresponding Prototype Temperatures at Thermocouple Location No. 3 and with Heater Power Input of 500 watts

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13 ABSTRACT Rules of similarity analysis are applied to develop criteria for deducing from model experiments steady-state and transient temperature distributions for a typical space vehicle element exposed to space conditions. The specific purpose is to determine if useful information can be obtained on models in space environment chambers. Thermal similarity criteria are deduced from expressions for transient conduction with radiation boundary conditions. Based on these criteria, a prototype and nominally one-half-scale model were designed and tested in a space chamber. To attain thermal similarity, some geometric distortion was necessary in the model. Both prototype and model were thermally cycled. Temperature data for prototype and model are compared at equivalent locations and times. Results show that temperatures agree within an average of approximately 1 percent with a maximum deviation of approximately 5 percent. The model was subsequently altered, thus destroying its thermal similarity. After alteration, the temperature data differed significantly from that of the thermally similar model and prototype. These experimental results demonstrate the correctness of the derived criteria and show that close adherence to similarity rules is required.

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